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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2016
SECOND YEAR [BATCH 2015-18]
MATHEMATICS FOR ECONOMICS [General]
Paper : III

Date : 23/12/2016
Time : 11 am – 2 pm
Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

Answer any four questions from Question No. 1 to 6 : [4×5]

1. a) State and prove Euler's theorem on homogeneous function for two variables. [1+1]
   b) Verify this theorem for the function: $u = \sin(xy)$. [3]

2. a) Define the Jacobian of some n variable functions $u_1, u_2, \cdots, u_n$ with respect to the variables $x_1, x_2, \cdots, x_n$. [2]
   b) Check whether the functions $u = x + y + z,\ v = xy + yz + zx,\ w = xyz$ are functionally related or not? If related then find the relation. [3]

3. a) Define stationary point of a function of two variable. [1]
   b) Show that $xy + \frac{8}{x} + \frac{8}{y}$ attains minimum value 12 at (2,2). [4]

4. Examine the convergence of $\int_a^b (x-a)^{-\frac{1}{2}}(b-x)^{-3}\,dx$. [5]

5. a) State the fundamental theorem of integral calculus. [2]
   b) Using the relation between Beta and Gamma function show that, $\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{3}{2}}\,dx = \frac{3\pi}{128}$. [3]

6. a) Define convexity and concavity of a function at a point. [1+1]
   b) Find the points of inflexion, if any of the curves
      (i) $y = \frac{x^3}{a^2 + x^2}$
      (ii) $x = (\log y)^3$ [1½+1½]

Answer any two questions from Question No. 7 to 9 : [2×10]

7. a) Show that for the function

$$f(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$$

$$f(0,y) = f(x,0) = 0$$

   $f_{xy} = f_{yx}$ at all points except (0,0). [6]

   b) Show that $\int_0^{\pi/2} \frac{x\,dx}{\sin x + \cos x} = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2}+1)$. [4]

8. a) Use the Lagrange's method of undetermined multipliers to find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$. [6]

b) Examine the convergence of $\int_0^1 \frac{x^{n-1}}{1-x}dx$. [4]

9. a) Prove that if $f(x, y)$ is differentiable at (a, b), it is continuous there. [3]

b) Is the converse of the above 9(a) is always true? Justify. [3]

c) Show that $f(x, y) = \sqrt{|xy|}$ is not differentiable at (0, 0). [4]

Group – B

Answer any seven questions from Question No. 10 to 20 : [7×5]

10. a) Find the differential equation of all circles of radius 'a' whose centres lie upon the y-axis. [3]

b) Find the order and degree of the following differential equation:

$$\left(\frac{dy}{dx}\right)^4 + 4\frac{d^2y}{dx^2} + \left(\frac{d^3y}{dx^3}\right)^2 = 0.$$ [2]

11. a) State the existence and uniqueness theorem for first order and first degree differential equation. [2]

b) Solve: $x^2 y dx - (x^3 + y^3)dy = 0$. [3]

12. Reduce the differential equation $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ to homogeneous form and then solve it. [5]

13. a) State the necessary and sufficient condition for a differential equation of first order and first degree to be exact. [2]

b) Check whether $(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0$ is an exact differential equation or not? Hence solve it. [3]

14. Prove that $\frac{1}{3x^3y^3}$ is an integrating factor of the differential equation $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$ also solve it. [2+3]

15. Solve: $(4x^2y - 6)dx + x^3dy = 0$. [5]

16. a) Define the Clairaut's form of a differential equation. [1]

b) Solve $y = px + \sqrt{a^2p^2 + b^2}$ where 'a' and 'b' are constants and $p \equiv \frac{dy}{dx}$. Also find the singular solution of the differential equation. [2+2]

17. Solve the differential equation $\frac{d^2x}{dt^2} + n^2x = 0$ when $t = 0$, $\frac{dx}{dt} = 0$ and $x = 0$. [5]

18. Solve: $\frac{d^2y}{dx^2} + y = 1 + 4x + 3x^2$. [5]

19. Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 2x^2 + 1$ by the method of undetermined coefficient. [5]

20. Solve the differential equation $\frac{d^2y}{dx^2} + a^2y = \sec ax$ by the method of variation of parameters. [5]

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